Quadric Surfaces

When teaching calculus I thought the book's treatment of quadric surfaces, while of course technically correct, left a little bit to be desired as far as user-friendliness. This is the beginner's guide I assembled as a supplement.

1. Ellipsoid (egg/sphere)

Standard equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

Identifying characteristics: This one's pretty straightforward – all three variables are squared and summed, with a non-zero constant on the other side of the equals sign.

Connecting it to its graph: The ellipsoid will meet each axis at the points $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$, and the negative of each of those.

2. Elliptic Paraboloid (satellite dish)

Standard equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z}{c}$

Identifying characteristics: The sum of two squared variables on one side of the equals sign, the third variable unsquared on the other. No extra constants roaming about.

Connecting it to its graph: The sign of the constant affiliated with the unsquared variable (in this case c) determines which direction the EP opens in. The unsquared variable tells which axis dives through the center of the EP. In this equation, the EP will open around the z axis, up (positive) if c is positive, and down if c is negative. The graph of the equation above contains the points $(a, 0, c)$, $(-a, 0, c)$, $(0, b, c)$, and $(0, -b, c)$. That is, there is an ellipse orthogonal to the axis the EP opens along, with radii coming from the constants affiliated with the two squared variables.

Other notes: If $a = b$, it is called a circular paraboloid.

3. Elliptic Cone (two megaphones)

Standard equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$

Identifying characteristics: The sum of two squared variables on one side of the equals sign, the third variable squared on the other side. No free agent constants.

Connecting it to its graph: The solo variable (in this case z) tells which axis the EC opens along, in both the positive and the negative directions. As for the elliptic paraboloid, we find ellipses c out from the origin, with size determined by a and b. The difference here is that with c squared, there is one in each direction.

Rebecca Weber, 2004

4. Hyperboloid of One Sheet (girdle)

Standard equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$

Identifying characteristics: All three variables are squared and on the same side of the equals sign. Two are summed and the third subtracted from them. There is a nonzero constant on the other side of the equation.

Connecting it to its graph: The axis indicated by the subtracted variable goes through the center of the one-sheeted H. Around the origin there is an ellipse with size given by the coefficients of the two positive terms. In the equation above, the one-sheet H intersects the x and y axes at $(a, 0, 0)$, $(0, b, 0)$ and their negatives, and does not intersect the z axis at all.

5. Hyperboloid of Two Sheets (two satellite dishes)

Standard equation: $\frac{z^2}{a^2} - \frac{x^2}{b^2} - \frac{y^2}{c^2} = 1$

Identifying characteristics: As for the hyperboloid of one sheet, all three variables are squared and on the same side of the equals sign, with a nonzero constant on the other. The difference here is that two of the terms are negative and one positive.

Connecting it to its graph: In this case the positive term gives the axis the two-sheet H opens along in both directions. The two pieces do not meet at the origin; each one starts c out from the origin. In the equation above, the vertices are at $(0, 0, c)$ and $(0, 0, -c)$. That should be enough to identify a graph.

6. Hyperbolic Paraboloid (saddle)

Standard equation: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = \frac{z}{c}$

Identifying characteristics: Two variables are squared, with their difference on one side of the equals sign. The third variable, unsquared, appears on the other side of the equals. No extraneous constants and no negatives besides the subtraction (c must be positive here).

Connecting it to its graph: The graph will go through the origin. There will be two parabolas also going through the origin, living in the planes that intersect at the axis of the unsquared variable. In the equation above, the parabolas lie in the xz and yz planes. If the unsquared variable were y, the parabolas would lie in the xy and yz planes. They each open along the axis of the unsquared variable, here z. The one that is in the plane also corresponding to the positive squared term opens in the positive direction, and the other in the negative direction. Here, since the y^2 is the positive term, the parabola in the yz plane will open in the positive z direction, and the parabola in the xz plane will open in the negative z direction.