Trigonometry for Calculus

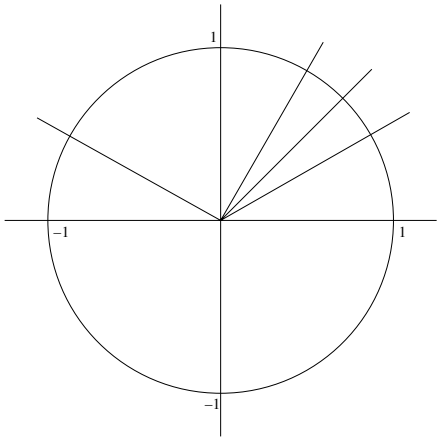
You need to know how the six trig functions relate to each other, the identity $\sin^2(x) + \cos^2(x) = 1$, and the sum and difference rules for sine and cosine. The following is meant as a mnemonic aid for calculating values of trig functions.

The important functions to know are sine and cosine. Everything else can be built off of them. The important values of $\sin(x)$ and $\cos(x)$ to know are for $x = 0$ and the integer multiples of $\pi/6$ and $\pi/4$. If straight memorization is easier for you, learn them that way. If, like me, you have difficulty with rote memorization, here is a suggestion you might try.

On the important values of x, sine and cosine give only a small number of outputs. In order of magnitude, they are $0, \frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}$, and 1; their negatives will also appear. Besides the output values themselves, here's what you have to remember:

(1) that $\frac{1}{\sqrt{2}}$ always goes with itself;
(2) that $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$ always go together;
(3) that 0 and 1 always go together;
(4) that cosine goes with x and sine goes with y;
(5) the unit circle.

In 1-3 note that the sign can vary; these are comments on the pairings of the magnitudes. The unit circle is for keeping track of what inputs the output values go with, which one of the pair goes with sine and which with cosine, and whether they are positive or negative. The following is a unit circle with axes, plus lines at angles of $\pi/6$, $\pi/4$, $\pi/3$, and $5\pi/6$.



The x and y values of where the line at angle θ intersects the unit circle are $\cos(\theta)$ and $\sin(\theta)$, respectively. The horizontal axis on the right hand side, an angle of 0, intersects the unit circle at the point (1, 0). Thus $\cos(0) = 1$ and $\sin(0) = 0$. At the vertical axis the opposite happens; $\cos(\pi/2) = 0$ and $\sin(\pi/2) = 1$. At the next line up, $\pi/6$, the x and y values are different but they are not 0 and 1. Therefore they must be $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$. Which one is x and which is y? Our x value is larger, so it must be $\frac{\sqrt{3}}{2}$, making $y = \frac{1}{2}$. Thus $\cos(\pi/6) = \frac{\sqrt{3}}{2}$ and $\sin(\pi/6) = \frac{1}{2}$. The opposite happens at the angle $\pi/3$; $\cos(\pi/3) = \frac{1}{2}$ and $\sin(\pi/3) = \frac{\sqrt{3}}{2}$. How about $\pi/4$? The x and y values are equal where that line intersects the unit circle, so the values must be $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$. Note that in the first quadrant all the x and y values (and thus the sine and cosine values) are positive.

Given that, let's find sine, cosine, and tangent for $5\pi/6$. The x and y values are different but not 0 and 1, so they must be $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$ or their negatives. Which has a larger magnitude? We're further away from the origin in the x direction than we are in the y direction, so we must have $|x| = \frac{\sqrt{3}}{2}$ and $|y| = \frac{1}{2}$. What about signs? We're still positive in the y direction, but we're negative in the x direction, so we end up with $\cos(5\pi/6) = -\frac{\sqrt{3}}{2}$ and $\sin(5\pi/6) = \frac{1}{2}$. Tangent is just sine over cosine, so $\tan(5\pi/6) = (\frac{1}{2})/(-\frac{\sqrt{3}}{2})$, which simplifies to $-\frac{1}{\sqrt{3}}$.

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