Curve Sketching

We have learned how to find a lot of information about a curve, all in bits and pieces. It's time to put it all together. Here's a list of the important aspects of a function we know how to find:

(1) the domain
(2) symmetry (odd and even functions)
(3) periodicity (for some fixed p, $f(x+p) = f(x)$ for all x, such as with trigonometric functions)
(4) the x- and y-intercepts
(5) local extreme values
(6) holes, places with removable discontinuities
(7) jumps, possible at the break point of a piecewise-defined function
(8) vertical asymptotes, places where $f(x)$ limits to $\pm\infty$ from either side
(9) intervals on which f is increasing or decreasing (delineated by critical points)
(10) intervals on which f is concave up or down (delineated by inflection points)
(11) horizontal asymptotes, what happens as x approaches $\pm\infty$
(12) slant asymptotes, where the limit at ∞ is ∞ but f approaches a particular line

Items 1, 2, and 3 are for your information – knowing them ahead of time can reduce the work involved in sketching the curve. The others are all pieces of the graph itself, necessary for a complete image of the function. Items 4 and 5 tell you specific points on the graph, as well as 8 if you find the y-values of the inflection points, and 6, 7, and 8 also give you information about specific x-values. Items 9 and 10 tell you how to fill in the blanks between those points. Items 11 and 12 concern the behavior of the graph as $|x|$ gets very large, outside all the values found in 4–8.

The General Method (a.k.a. the way I do it)

(1) Consider the domain of the function $f(x)$ and, if they are readily apparent, the symmetry and periodicity.
(2) Find the x- and y-intercepts of f and plot them.
(3) Find any places where the denominator of f is zero and decide whether they are holes or asymptotes. For holes, find the value of f and plot the hole. For vertical asymptotes, decide whether f approaches positive or negative ∞ on each side. Sketch in roughly.
(4) Look for jumps in f and plot them (only in piecewise-defined f).
(5) Find $f'(x)$ and the critical points of f.
(6) Determine whether each critical point is a local max or min (or neither), and whether the function is increasing or decreasing between them. Plot the local extreme values.
(7) Find $f''(x)$ and the possible inflection points of f.
(8) Determine whether the possible inflection points are actually inflection points, and determine whether f is concave up or down in between them.
(9) Use 6 and 8 to fill in the blanks between all the previously-plotted points.
(10) Determine whether f has horizontal or slant asymptotes. Use this information plus 6 and 8 to draw f's behavior as x approaches $\pm\infty$.

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